Ejercicios Mecánica Teórica. Capítulo 34

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1. Calcular $T^{\mu\nu}$

Definiendo el tensor energía-momento como

$$T^{\mu\nu} = \frac{\partial \mathscr{L}}{\partial \left(\partial_\mu \phi\right)} \partial^\nu \phi - g^{\mu\nu} \mathscr{L} \tag{1}$$

Y sea el Lagrangiano

$$\mathscr{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\phi) \tag{2}$$

Lo primero vamos a calcular la derivada

$$\frac{\partial \mathscr{L}}{\partial \left(\partial_\mu \phi\right)} = \frac{1}{2} \frac{\partial \left(\partial_\nu \phi \partial^\nu \phi\right)}{\partial \left(\partial_\mu \phi\right)} = \frac{g^{\nu\alpha}}{2} \frac{\partial \left(\partial_\nu \phi \partial_\alpha \phi\right)}{\partial \left(\partial_\mu \phi\right)} = \frac{g^{\nu\alpha}}{2} \left(\delta^\mu_\nu \partial_\alpha \phi + \partial_\nu \phi \delta^\mu_\alpha\right) = \frac{1}{2} \left(\partial^\mu \phi + \partial^\mu \phi\right) = \partial^\mu \phi \tag{3}$$

Por lo que, para este Lagrangiano concreto tenemos

$$T^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \mathscr{L} \tag{4}$$

Con esto ya vemos que, evidentemente T es un tensor simétrico, calculando las componentes tenemos

$$T^{11} = \frac{1}{2} \left(\dot{\phi}^2 + (\partial_x \phi)^2 + (\partial_y \phi)^2 + (\partial_z \phi)^2\right) + U(\phi) \tag{5}$$

$$T^{01} = -\dot{\phi} \partial_x \phi = T^{10} \tag{6}$$

$$T^{02} = -\dot{\phi} \partial_y \phi = T^{20} \tag{7}$$

$$T^{03} = -\dot{\phi} \partial_z \phi = T^{30} \tag{8}$$

$$T^{11} = \frac{1}{2} \left(\dot{\phi}^2 + (\partial_x \phi)^2 - (\partial_y \phi)^2 - (\partial_z \phi)^2\right) - U(\phi) \tag{9}$$

$$T^{12} = \partial_x \phi \partial_y \phi = T^{21} \tag{10}$$

$$T^{13} = \partial_x \phi \partial_z \phi = T^{31} \tag{11}$$

$$T^{22} = \frac{1}{2} \left(\dot{\phi}^2 - (\partial_x \phi)^2 + (\partial_y \phi)^2 - (\partial_z \phi)^2\right) - U(\phi) \tag{12}$$

$$T^{23} = \partial_y \phi \partial_z \phi = T^{32} \tag{13}$$

$$T^{33} = \frac{1}{2} \left(\dot{\phi}^2 - (\partial_x \phi)^2 - (\partial_y \phi)^2 + (\partial_z \phi)^2\right) - U(\phi) \tag{14}$$